Universal Statistical Science

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Measurement data processing is based on classical probability theory with its defects and on mathematical statistics [1]. It mixes equiprecise a priori but nonequiprecise data a posteriori without adequately weighting. Finite samples with skew distributions are given predefined infinite symmetric distributions with proper approximation near maximums only. Using the quadratic mean error and the arithmetic mean m (as the expectation) only leads to the illusion that this mean is the most probable value also by strongly asymmetric distributions with great skewness. Classical statistics either overestimates or subjectively ignores outliers and leads to results instability. Regression analysis gives the least square method results different by mutually replacing the variables roles and only qualitatively proves the fact of dependence via rejecting the null hypothesis (full independence) without estimating errors and improving formulae. Tabular data only provide neither analytical nor computational operability. There is no polymethodicity with results comparison and adequacy test.

Universal statistical science [2, 3] uses base sign conserving exponentiation $a^{''b} = |a|^b$ sign a for negative bases in universal mathematics. Moments and absolute moments

$$^{s|t}M_y = \Sigma_{i=1}^n w_i^s (x_i - y)^{''t} / \Sigma_{i=1}^n w_i^s, \quad ^{s|t}|M|_y = \Sigma_{i=1}^n w_i^s |x_i - y|^t / \Sigma_{i=1}^n w_i^s$$

of any (possibly noninteger) orders s and t with adequately weighting a posteriori equiprecise data a priori x_i with positive weights w_i where y may be, e.g., exact value x_0 , mean m , or unimode u for which $\Sigma_{i=1}^n w_i (x_i - u)^{''t} = 0$ are very efficient.

Signed power mean and s|t-standard deviation of a single data by known x_0 are

$$^{s|t}m_y = y + [\Sigma_{i=1}^n w_i^s (x_i - y)^{''t} / \Sigma_{i=1}^n w_i^s]^{''1/t}, \quad ^{s|t}\sigma_x = [\Sigma_{i=1}^n w_i^s |x_i - x_0|^t / \Sigma_{i=1}^n w_i^s]^{1/t}.$$

Properly weighting asymmetric sample distributions with introduced asymmetry grade $A = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$ can be based on different left and right standard deviations σ_L and σ_R by values $x_1 \le x_2 \le \dots \le x_n$ or their numbers. Also to asymmetric populations apply, e.g., the weights corresponding to binormal, biarctan, or binomial distributions

$$w_i = \exp[-(x_i - u)^2/(2\sigma_L^2)],\ \sigma_L^2 = \Sigma_{x(i)\le u} (x_i - u)^2/\Sigma_{x(i)\le u} 1 \text{ by } x_i \le u ,$$
$$w_i = \exp[-(x_i - u)^2/(2\sigma_R^2)],\ \sigma_R^2 = \Sigma_{x(i)\ge u} (x_i - u)^2/\Sigma_{x(i)\ge u} 1 \text{ by } x_i \ge u ;$$
$$w_i = 1/[1 + (x_i - u)^2/(3\sigma_L^2)] \text{ by } x_i \le u ,\ w_i = 1/[1 + (x_i - u)^2/(3\sigma_R^2)] \text{ by } x_i \ge u ;$$
$$w_i = (n-1)!/[\Gamma(h)\Gamma(n - h + 1)]\, p^{h-1}(1 - p)^{n-h},\ h(i) = 1 + (n-1)(x_i-x_1)/(x_n-x_1),\ p=(h_{med}-1)/(n-1),$$
$$w_i = (n - 1)!/[(i - 1)!(n - i)]\, p^{i-1}(1 - p)^{n-i}, \text{ or simply } w_i = (n - 1)!/[(i - 1)!(n - i)].$$

Universal statistical science applies to the classical Cavendish data [4] (Figures 1, 2).

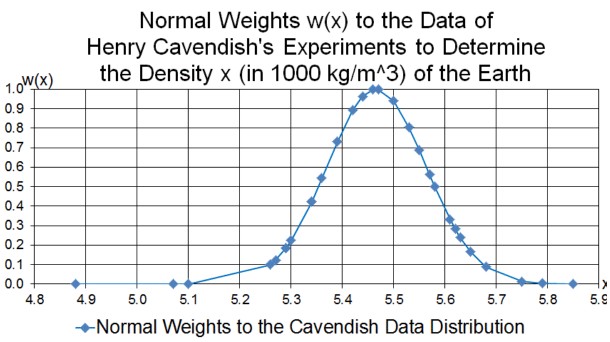

Figure 1. Weights to Cavendish's data

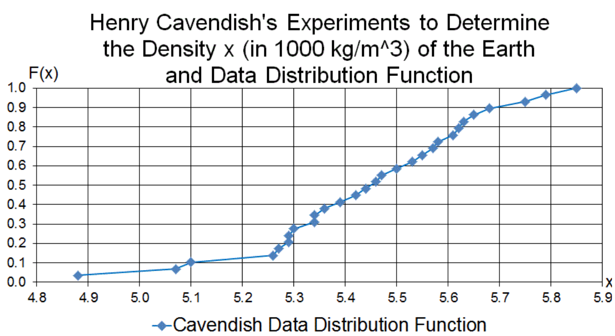

Fig. 2. Cavendish's data distribution F(x)

The modern value for the Earth density in 10^3 kg/m^3 is ρ = 5.51 and for the Newtonian constant of gravitation in 10^{-11} m^3 kg^{-1} s^{-2} G = 6.674 based on the reported

data between 6.672 and 6.676 [5]. Using Cavendish's 29 data, classical statistics gives $\rho = m = 5.448$ with standard deviation $\sigma_\rho = 0.22$ and $G = 6.752$; universal statistical science gives $\rho = u = 5.466$ with deviations $^{u}\sigma_L = 0.096$, $^{u}\sigma_R = 0.097$, and $G = 6.729$. Using Cavendish's 23 data after changing a wire via a stiffer wire, classical statistics gives $\rho = m = 5.483$ with standard deviation $\sigma_\rho = 0.19$ and $G = 6.708$ but using median $m_{ed} = 5.46$ and following Charlier [1] $u = 3m_{ed} - 2m = 5.414$ and $G = 6.794$ whereas universal statistical science gives $\rho = u = 5.49$ with deviations $^{u}\sigma_L = 0.10$, $^{u}\sigma_R = 0.12$, $G = 6.700$. Therefore, applying universal statistical science to the classical Cavendish data gives new results for the Newtonian constant of gravitation.
Universal statistical science provides scientific fundament for universal data processing science. Their theories and methods of setting and solving many typical urgent problems including data processing even by great data scatter, asymmetry, skewness, and outliers give adequate and correct results, e.g. in aeronautical fatigue.

Keywords: Ph. D. & Dr. Sc. Lev Gelimson, "Collegium" All World Academy of Sciences, Academic Institute for Creating Fundamental Sciences, Mathematical Journal, universal statistical science, mathematical statistics, null hypothesis, polymethodicity, analytical operability, computational operability, Universal Probabilistic Science, Measurement Science, measurement data processing, outlier, classical probability theory, at most countably additive probability, impossible event, certain event, formal derivative sense, maximum likelihood principle, commeasure, mixed distribution, physical and mathematical modeling and measurement information, losing physical sense, probabilistic and statistical analysis, regression analysis, strongly asymmetric distribution, great skewness, uniquantity, even uncountably algebraically additive measure with universal conservation law, universal point measure, infinite cardinal number, continuum cardinality, perfectly sensitive universal measure, strictly positive uninumber probability, both mathematical and physical sense of classical probability density, infinitesimal, real number, fuzzy set, multiset, countable set, concrete mixed physical magnitude, absolute error, relative error, artificially limiting precision, ignoring systematic error, precision illusion, weighting a posteriori possibly equiprecise data a priori, quadratic mean error, Legendre, Gauss, universal mathematics, unimathematics, Unimathematik, universal physics, material unistrength, invariant and universal dimensionless physical quantity, ion implantation unidose, mechanical unistress, discovering universal strength laws of nature, anisotropic material, different resistance to tension and compression, variable load, rotating the principal directions of the stress state at a point, reliable measurement data, great data scatter, uniarithmetics, quantialgebra, quantianalysis, quantioperation, quantirelation, canonic set, canonic positive infinity, signed zero reciprocal, canonic overinfinity, algebraically quantioperable quantielement, quantioperation, quantiset, multiquantity, unierror, unireserve, unireliability, unirisk, exactness confidence, eliminating averaging and partition error, temporary overprecision, instrument precision, noninteger-order moment, infinite moment, variance, predefined distribution, normal distribution, binomial distribution, suitably operable pointwise probability distribution function, binormal probability density, biarctan probability density, sign-conserving multiplication, base sign conserving exponentiation, a posteriori weighting a priori equiprecise data, absolute moment, analytizing tabular function, standard deviation, properly weighting asymmetric sample distribution, asymmetry grade, asymmetric population, universal data processing science, Henry Cavendish, Experiments to Determine the Density of the Earth, Earth density, Newtonian constant of gravitation, Fundamental Physical Constant, median, Charlier, aeronautical fatigue, Cramér, general problem theory,

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