Universal Probabilistic Science

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Measurement data processing is based on classical probability theory [1]. It simply postulates the existence of at most countably additive probability between 0 for impossible events and 1 for certain events. But probability $p_n = p$ of selecting certain $n \in N = \{1, 2, ...\}$ does not exist. In any continual set, the probabilities to take distinct values are regarded as the same (zero) but with different probability densities and no possibility to obtain required 1 as the sum. Probability densities f(x) with formal derivative sense can exceed 1 and are no probabilities. Then the maximum likelihood principle has no clear sense. There are no perfectly sensitive universal measure with conservation laws and no commeasure for discrete and continual parts of a mixed distribution. The quadratic mean error leads to the illusion that the arithmetic mean is the most probable value by strongly asymmetric distributions with great skewness. Noninteger-order and infinite moments do not exist at all. The variance, higher-order moments, and the least square method too strongly magnify the inputs of data with great errors by reducing the roles of the most precise data. Real samples are given predefined distributions. The typical normal distribution symmetrizes and infinitizes the binomial distribution with adequately fitting it near its maximum and p = 0.5 only.

Universal mathematics [2-5] provides the uninumbers. In them, universal probabilistic science gives $p_n = p = 1/\omega$ (the first Cantorian cardinal $\aleph_0$), and an n-dimensional probability with probability density f(x) is simply $f(x)/\Omega^n$ (Ω the continuum cardinality $\mathbb{C}$). In the axioms system, a random variable is a quantiset of values with uncountably additive nonnegative uninumber quantities which are pointwise probabilities to take these values and with unit uniquantity. Possibly uncountably quantiunifying random variables uses unifying their sets of values with averaging their probabilistic quantities of the same values. Suitably operable pointwise probability distribution functions g(x) provide adequately regarding typical sample distribution skewness (Figures 1 and 2).

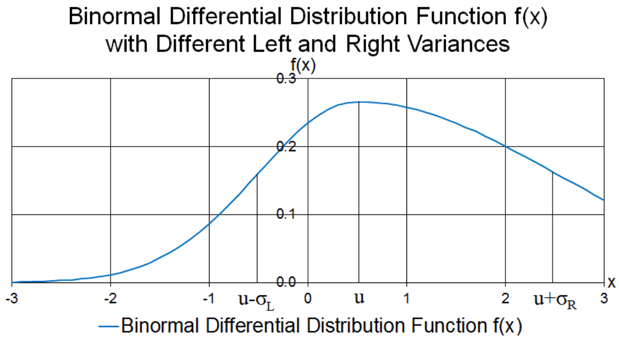

Figure 1. The binormal probability density

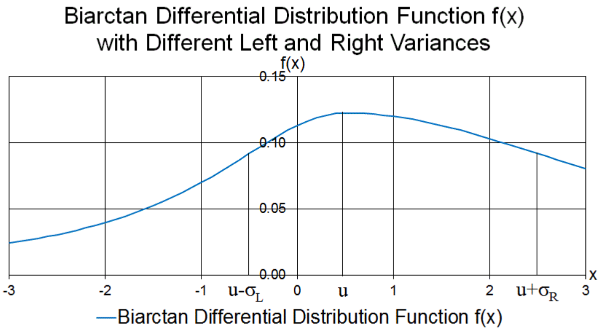

Figure 2. The biarctan probability density

Apply the maximum likelihood principle to $f(x) = \Omega g(x)$ by possibly asymmetric sample distributions of random variables (and their values) $x_1 \le x_2 \le \dots \le x_n$ with generally different left and right variances σ_{Li}^2 and σ_{Ri}^2 via $f_i(x)$, as well as σ_L^2 and σ_R^2 for total f(x) asymmetric even by symmetric $f_i(x)$ ($i = 1, 2, \dots, n$). Quantiunifying binormal distributions provides general analytical solution giving by $\sigma_{Li}^2 = \sigma_{Ri}^2$ unimode u via

$$u = \Sigma_1^n x_i/n - \Sigma_{x(i)\le u} (x_i-u)\{1 - \exp[-(x_i-u)^2/(2\sigma_L^2)]\}/n - \Sigma_{x(i)\ge u} (x_i-u)\{1-\exp[-(x_i-u)^2/(2\sigma_R^2)]\}/n,$$

$$\sigma_L^2 = \Sigma_{x(i)\le u} (x_i - u)^2 \exp[-(x_i - u)^2/(2\sigma_L^2)] / \Sigma_{x(i)\le u} \exp[-(x_i - u)^2/(2\sigma_L^2)],$$

$$\sigma_R^2 = \Sigma_{x(i)\ge u} (x_i - u)^2 \exp[-(x_i - u)^2/(2\sigma_R^2)] / \Sigma_{x(i)\ge u} \exp[-(x_i - u)^2/(2\sigma_R^2)]$$

using intelligent iteration. Alternative sign-conserving multiplication "$\Pi_{j\in J}$ a_j = min(sign a_j | j ∈ J) |$\Pi_{j\in J}$ a_j| provides alternative base sign conserving exponentiation a"^{b} = |a|^b sign a for negative bases and hence unrestricted power and exponential functions, e.g. moments of any (e.g. noninteger) orders. Universal probabilistic science includes general theories of efficiently utilizing a posteriori weighting a priori equiprecise data, of asymmetrizing classical and introduced distributions, of efficiently determining unimodes with really maximum probabilities, and of analytizing tabular functions.
Universal probabilistic science provides scientific fundament for universal statistical science and universal data processing science. Their theories and methods of setting and solving many typical urgent problems including data processing even by great data scatter give adequate and correct results, e.g. in aeronautical fatigue.

Keywords: Ph. D. & Dr. Sc. Lev Gelimson, "Collegium" All World Academy of Sciences, Academic Institute for Creating Fundamental Sciences, Mathematical Journal, Universal Probabilistic Science, Measurement Science, measurement data processing, classical probability theory, at most countably additive probability, impossible event, certain event, formal derivative sense, maximum likelihood principle, commeasure, mixed distribution, physical and mathematical modeling and measurement information, losing physical sense, probabilistic and statistical analysis, strongly asymmetric distribution, great skewness, uniquantity, even uncountably algebraically additive measure with universal conservation law, universal point measure, infinite cardinal number, continuum cardinality, perfectly sensitive universal measure, strictly positive uninumber probability, both mathematical and physical sense of classical probability density, infinitesimal, real number, fuzzy set, multiset, countable set, concrete mixed physical magnitude, absolute error, relative error, artificially limiting precision, ignoring systematic error, precision illusion, weighting a posteriori possibly equiprecise data a priori, quadratic mean error, Legendre, Gauss, universal mathematics, unimathematics, Unimathematik, universal physics, material unistrength, invariant and universal dimensionless physical quantity, ion implantation unidose, mechanical unistress, discovering universal strength laws of nature, anisotropic material, different resistance to tension and compression, variable load, rotating the principal directions of the stress state at a point, reliable measurement data, great data scatter, uniarithmetics, quantialgebra, quantianalysis, quantioperation, quantirelation, canonic set, canonic positive infinity, signed zero reciprocal, canonic overinfinity, algebraically quantioperable quantielement, quantioperation, quantiset, multiquantity, unierror, unireserve, unireliability, unirisk, exactness confidence, eliminating averaging and partition error, temporary overprecision, instrument precision, noninteger-order moment, infinite moment, variance, predefined distribution, normal distribution, binomial distribution, suitably operable pointwise probability distribution function, binormal probability density, biarctan probability density, sign-conserving multiplication, base sign conserving exponentiation, a posteriori weighting a priori equiprecise data, analytizing tabular function, universal statistical science, universal data processing science, aeronautical fatigue, general problem theory, elastic mathematics, general strength theory, corrections and generalizations of the least square method.

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