Deformable Solid Unimechanics

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Deformable solid mechanics [1-3] uses nonuniversal dimensional mechanical stresses without expressing their risk (danger) degrees. There are no known simple namely analytical solutions to nontrivial truly three-dimensional problems without the relative smallness of some characteristic solids sizes such as thickness even in thick plate theory. There are no known general power-law solutions to the homogeneous harmonic and biharmonic equations playing key roles not only in elasticity theory. Analytically testing the results of the finite element method (FEM) etc. is necessary.

Deformable solid unimechanics (DSUM) [4-6] defines the universal stresses, or unistresses. Analytical uniparametrization science includes many general analytical theories and methods useful for solving uniproblems. General uniparametrization theory searches for a general solution to a uniproblem in its general pseudosolutions. General (possibly infinite) linear-combination theory provides explicitly determining the general solution to a uniproblem as a unisystem of equations and naturally generalizes the classical definition of a finite linear independence to infinities. To the homogeneous biharmonic equation $\nabla^2\nabla^2 L(r, z) = 0$ over desired function $L(r, z)$ in the cylindrical coordinates, r, z, in the class of power functions, or, equivalently, in a general pseudosolution $L(r, z) = \Sigma_{i=0}^{\infty}\Sigma_{j=0}^{\infty} a_{ij} r^{2i} z^j$, the most general solution is found:

$$L(r, z) = \Sigma_{i=0}^{\infty}\Sigma_{j=0}^{\infty}(-1)^{i+1} i!^{-2} j!^{-1}[(2i + j - 2)!\, i\, 2^{2-2i} a_{1,\, 2i+j-2} + (2i + j)!(i - 1)2^{-2i} a_{0,\, 2i+j}] r^{2i} z^j$$

where $k! = 1$ and $a_{1k} = 0$ by $k < 0$; a_{0j} and a_{1j} are two arbitrary number sequences.

See displacements u_r, u_z (mm), stresses σ_r, σ_t, σ_z, τ_{rz} (MPa) in a uniformly supported below (30 mm ≤ r ≤ 55 mm) cylindrical glass element with radius 55 mm and height 60 mm under above and side pressure 98 MPa via DSUM (Fig. 1) vs. FEM (Fig. 2):

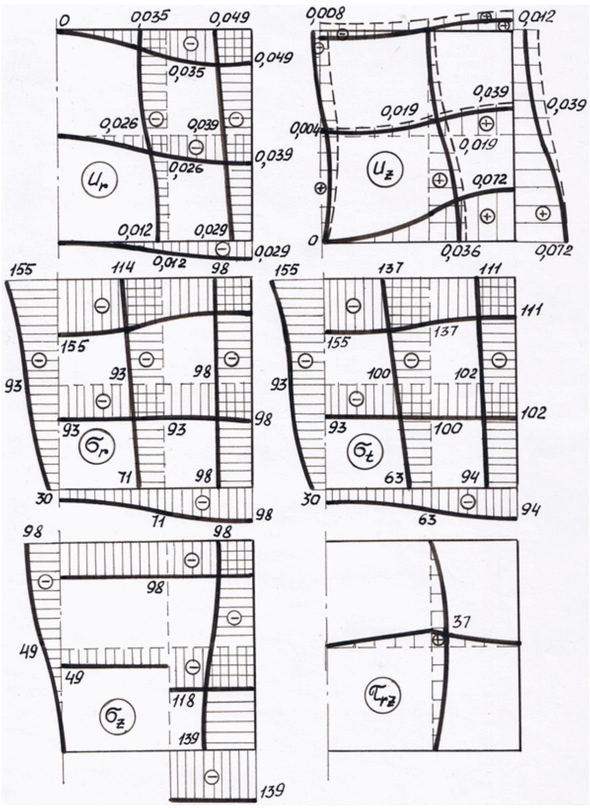

Fig.1. DSUM Displacements and stresses

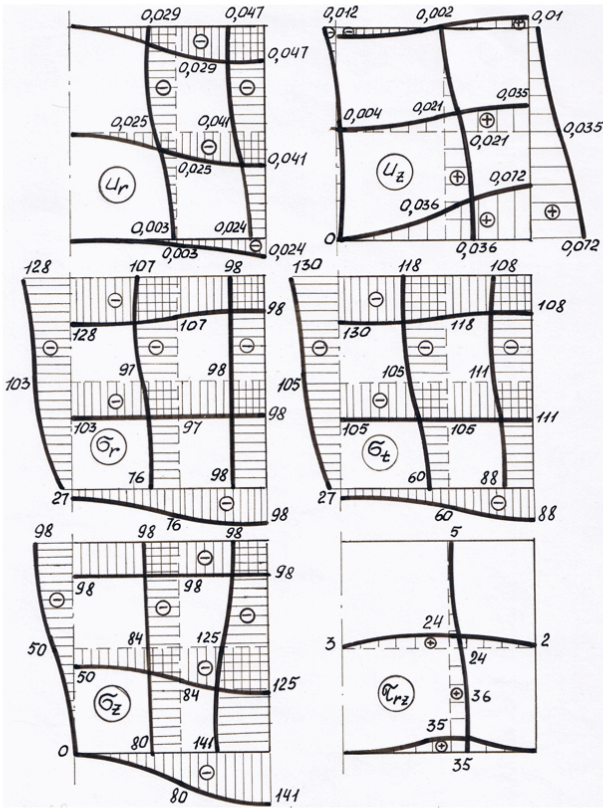

Fig. 2. FEM Displacements and stresses

To the homogeneous harmonic equation $\nabla^2\varphi(x, y, z) = 0$ over desired function $\varphi(x, y, z)$ in the Cartesian coordinates, x, y, z, in the class of power functions, or in a general pseudosolution $\varphi(x, y, z) = \Sigma_{i=0}^{\infty}\Sigma_{j=0}^{\infty}\Sigma_{k=0}^{\infty}a_{ijk}x^iy^jz^k$, the most general solution is

$$\varphi(x, y, z) = \Sigma_{i=0}^{\infty}\Sigma_{j=0}^{\infty}\Sigma_{k=0}^{\infty}(-1)^{[i/2]}[i/2]!(i!j!k!)^{-1}\Sigma_{m=0}^{[i/2]}(j+2m)!(k+2[i/2]-2m)!(m!([i/2]-m)!)^{-1}\times$$

$$a_{i-2[i/2],\, j+2m,\, k+2[i/2]-2m}x^iy^jz^k$$ where $[b] = \mathrm{floor}(b)$; a_{0jk} and a_{1jk} are any number sequences.
Deformable solid unimechanics (also including analytical unirestructuring science, power and integral analytic macroelement sciences) solves earlier principally unsolvable nontrivial truly three-dimensional problems, e.g., in aeronautical fatigue.

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[1] Gabriel Lamé. Lecons sur la theorie mathematique de l'élasticite des corps solides. Gauthier-Villars, Paris, 1852

[2] Augustus Edward Hough Love. A Treatise on the Mathematical Theory of Elasticity. Vols. I, II. Cambridge University Press, Cambridge, 1892, 1893

[3] Stephen Timoshenko. Theory of Elasticity. 3rd ed. McGraw-Hill, New York, 1970

[4] Lev Gelimson. Elastic Mathematics. General Strength Theory. The "Collegium" All World Academy of Sciences Publishers, Munich, 2004

[5] Lev Gelimson. Linear Combination Method in Three-Dimensional Elasticity. In: Review of Aeronautical Fatigue Investigations in Germany during the Period May 2007 to April 2009, Ed. Dr. Claudio Dalle Donne, Pascal Vermeer, CTO/IW/MS-2009-076 Technical Report, International Committee on Aeronautical Fatigue, ICAF 2009, EADS Innovation Works Germany, 2009, 38-39

[6] Lev Gelimson. Analytic Macroelement Method in Axially Symmetric Elasticity. In: Review of Aeronautical Fatigue Investigations in Germany during the Period May

2007 to April 2009, Ed. Dr. Claudio Dalle Donne, Pascal Vermeer, CTO/IW/MS-2009-076 Technical Report, International Committee on Aeronautical Fatigue, ICAF 2009, EADS Innovation Works Germany, 2009, 39-40