Advanced Unimathematics (Universal Mathematics)

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Classical mathematics [1] defines power and exponential functions for bases $a \geq 0$ only. Otherwise, raising is well-defined for even positive integer exponents only, see

$$(-1)^3 = -1 \neq 1 = [(-1)^6]^{1/2} = (-1)^{6/2},\ (-1)^{1/3} = -1 \neq 1 = [(-1)^2]^{1/6} = (-1)^{2/6}.$$

Exponentiation and further hyperoperations are noncommutative and nonassociative:

$$2^3 = 8 \neq 9 = 3^2,\ 2\wedge 3\wedge 4 = 2\wedge(3\wedge 4) = 2^{81} \neq 2^{12} = (2\wedge 3)\wedge 4.$$

Also iterated (nested) power-exponential functions (power towers), e.g., $y = x^x = {}^2x$, $y = {}^nx = x\wedge x\wedge ...\wedge x$ (n times), and $y = {}^xx$, are useful by $x \geq 1$ only.

Advanced universal mathematics [2, 3] has introduced alternative negativity-conserving multiplication $"\Pi_{j\in J}\ a_j = \min(\text{sign } a_j \mid j \in J)\ |\Pi_{j\in J}\ a_j|$ and base-sign-conserving exponentiation $a"^b = |a|^b \text{ sign } a$ with extending to complex $a = re^{i\varphi}$, $b = c + di$ ($i^2 = -1$):

$$a"^b = a"^{c+di} = |a|^{c+di} \text{ dir } a = r^{c+di} e^{i\varphi} = r^c r^{di} e^{i\varphi} = r^c e^{id \ln r} e^{i\varphi} = r^c e^{i(d \ln r + \varphi)}\ ;\ [(-1)"^6]"^{1/2} = -1.$$

Tetration having possibly noninteger multiplicity with $a > 0$ and x used $[a] + 1$ times

$$y = f(x) = {}^ax = x\wedge\wedge a = x^{\wedge 2} a = \exp_x^{[a]+1}(\{a\}) = x\wedge x\wedge ...\wedge x\wedge\{a\},\ [a] = \text{floor}(a),\ \{a\} = a - [a].$$

Notation: $a^{b\backslash c\backslash d} = a\wedge b\wedge c\wedge d$; $a° = \text{sign } a$; $a? = \max(a\ ,\ 1/a)$.

Transforming ${}^nx = x\wedge\wedge n$: $y = f(x) = {}^{n"}x = x"\wedge\wedge n = x°|x|\wedge\wedge^{n-1}|x|?$, $f(0) = 0$ (see Fig. 1, n = x).

Quanti-hyper-root-logarithm y = lh2\x inverse to power-exponential function y = x"^^2, quanti-hyper-root-logarithm y = lha\x inverse to y = x"^^a, and self-hyper-root-logarithm y = lh x inverse to y = x"^^x = (sign x) |x|^max(|x|, 1/|x|)^^(x - 1) (see Fig. 2).

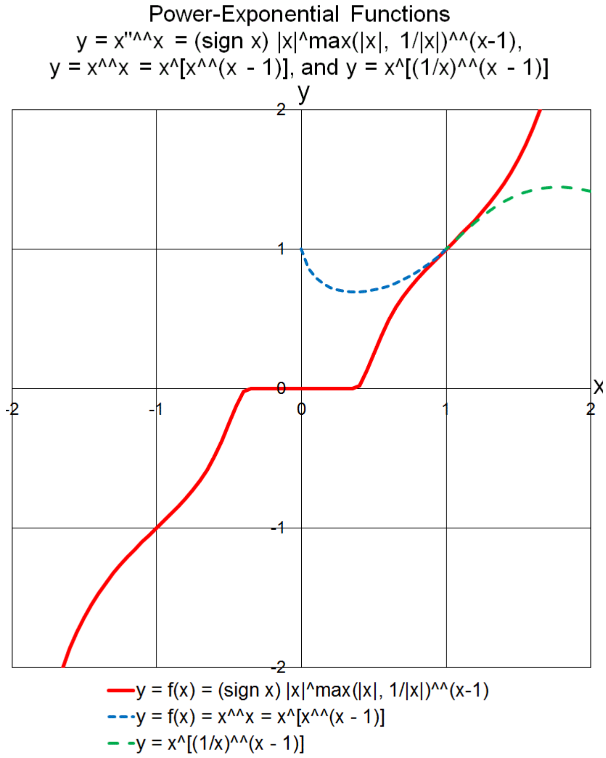

Fig. 1. Transformation useful everywhere

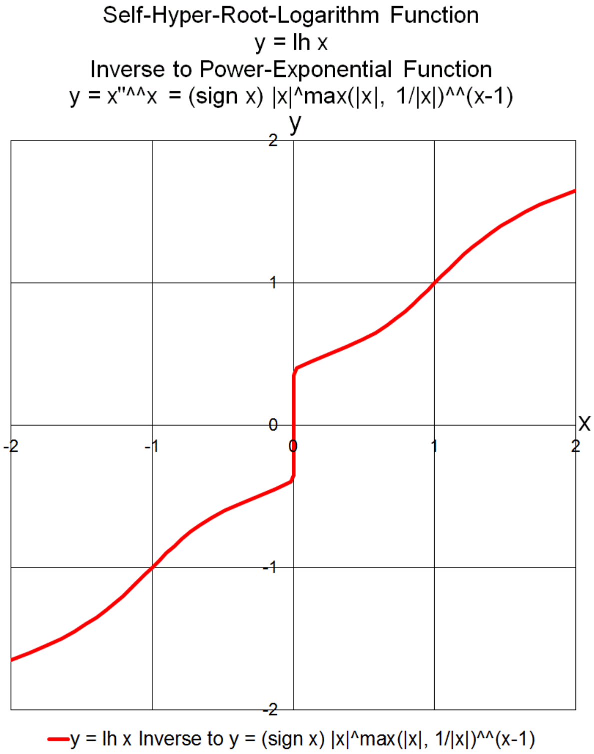

Figure 2. Transformation inversion

Power-sum exponentiation: $E\Sigma_{j\in J}\ a_j = \wedge^{+}{}_{j\in J}\ a_j$; $a \wedge^{+} b = a^b + b^a$; $2 \wedge^{+} 3 = 2^3 + 3^2 = 17$.

Power-modulus-sum exponentiation: $|E|\Sigma_{j\in J}\ a_j = {}^{|\wedge|+}{}_{j\in J}\ a_j$; $a\ {}^{|\wedge|+}\ b = |a^b| + |b^a|$.

Power-sum-modulus exponentiation: $E|\Sigma|_{j\in J}\ a_j = |E\Sigma_{j\in J}\ a_j| = \wedge^{|+|}{}_{j\in J}\ a_j$; $a \wedge^{|+|} b = |a^b + b^a|$.

Modulus-power-sum exponentiation: $E\Sigma_{j\in J}\ |a_j| = {}^{||\wedge+}{}_{j\in J}\ a_j$; $a^{||\wedge+}b = |a|\wedge^{+}|b| = |a|^{|b|} + |b|^{|a|}$.

Sign-power-modulus-sum exponentiation: $°|E|\Sigma_{j\in J}\ a_j = {}^{°|\wedge|+}{}_{j\in J}\ a_j$; $a^{°|\wedge|+}\ b = a°|a^b|+b°|b^a|$.

Sign-power-sum-modulus exponentiation: $^\circ|E\Sigma|_{j\in J}\, a_j = {}^{\circ|\wedge|+}{}_{j\in J}\, a_j$; $a^{\circ|\wedge+|}b=|a^\circ|a^b|+b^\circ|b^a||$.
Sign-modulus-power-sum exponentiation: $^\circ||E\Sigma_{j\in J}\, a_j = {}^{\circ||\wedge+}{}_{j\in J}\, a_j$; $a^{\circ||\wedge+}b=a^\circ|a|^{|b|}+b^\circ|b|^{|a|}$.
Modulus-sign-power-sum exponentiation: $|^\circ E\Sigma|_{j\in J}\, a_j = {}^{|\circ\wedge+|}{}_{j\in J}\, a_j$; $a^{|\circ\wedge+|}b=|a^\circ|a|^{|b|}+b^\circ|b|^{|a|}|$.
Power-sum-modulus-sign exponentiation: $|E\Sigma|^\circ{}_{j\in J}\, a_j = {}^{|\wedge+|\circ}{}_{j\in J}\, a_j$; $a^{|\wedge+|\circ}b = (a+b)^\circ|a^b+b^a|$.
Power-modulus-sum-sign exponentiation: $|E|\Sigma^\circ{}_{j\in J}\, a_j ={}^{|\wedge|+\circ}{}_{j\in J}a_j$; $a^{|\wedge|+\circ}b=(a+b)^\circ(|a^b|+|b^a|)$.
Modulus-power-sum-sign exponentiation: $||E\Sigma^\circ{}_{j\in J}a_j={}^{||\wedge+\circ}{}_{j\in J}a_j$; $a^{||\wedge+\circ}b=(a+b)^\circ(|a|^{|b|}+|b|^{|a|})$.
Power-sum maximum-exponentiation: $E^?\Sigma_{j\in J}\, a_j = {}^{?\wedge+}{}_{j\in J}\, a_j$; $a\ {}^{?\wedge+}\ b = a^{b?} + b^{a?}$.
Power-modulus-sum maximum-exponentiation: $|E|^?\Sigma_{j\in J}\, a_j = {}^{?|\wedge|+}{}_{j\in J}\, a_j$; $a^{?|\wedge|+}b=|a^{b?}|+|b^{a?}|$.
Power-sum-modulus maximum-exponentiation: $E^?|\Sigma|_{j\in J}\, a_j = {}^{?\wedge|+|}{}_{j\in J}\, a_j$; $a^{?\wedge|+|}b=|a^{b?}+b^{a?}|$.
Modulus-power-sum maximum-exponentiation: $E^{||?}\Sigma_{j\in J}\, a_j = {}^{||?\wedge+}{}_{j\in J}a_j$; $a^{||?\wedge+}b=|a|^{|b|?}+|b|^{|a|?}$.
Sign-modulus-power-sum maximum-exponentiation:$E^{\circ||?}\Sigma_{j\in J}a_j$;$a^{\circ||?\wedge+}b=a^\circ|a|^{|b|?}+b^\circ|b|^{|a|?}$.
Sign-power-sum-modulus maximum-exponentiation: $a\ {}^{\circ||?\wedge|+|}\ b = |a^\circ|a|^{|b|?} + b^\circ|b|^{|a|?}|$.
Modulus-power-sum-sign maximum-exponentiation: $a\ {}^{||?\wedge+\circ}\ b = (a + b)^\circ(|a|^{|b|?} + |b|^{|a|?})$.
Power-product exponentiation: $E\Pi_{j\in J}\, a_j = {}^{\wedge\times}{}_{j\in J}\, a_j$; $a\ {}^{\wedge\times}\ b$ = a^b b^a = $a^b\, b^a$.
Modulus-power-product exponentiation: $E\Pi_{j\in J}\, |a_j| = {}^{||\wedge\times}{}_{j\in J}\, a_j$; $a\ {}^{||\wedge\times}\ b = |a|^{|b|}\, |b|^{|a|}$.

Advanced unimathematics creates fundamentally new opportunities to set and solve many earlier principally unsolvable urgent problems, e.g., in aeronautical fatigue.

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[1] Encyclopaedia of Mathematics. Ed. Michiel Hazewinkel. Volumes 1 to 10. Supplements I to III. Kluwer Academic Publ., Dordrecht, 1987-2002
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